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PLUS★WARE TREASURE CHEST

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Complex Numbers

Release 6 of the APL★PLUS System for the PC includes a new utility workspace *COMPLEX* that helps you work with complex numbers. Complex numbers are represented to the utilities as pairs of real numbers. For example, a two-element numeric vector represents a complex scalar, a two-row matrix represents a complex vector, and so on. For convenience, a numeric scalar like 5 can be used as the complex scalar 5 0, the complex number with real part equal to 5 and imaginary part zero.

Complex arrays are formed with APL primitives or a variety of functions in the workspace, the simplest being

J— Pairs corresponding real and imaginary parts

D—Pairs corresponding magnitudes and degrees

R—Pairs corresponding magnitudes and radians

For example, the integer units can be generated by

```
   1 0 ¯1 0 J 0 1 0 ¯1
1 0 ¯1  0
0 1  0 ¯1
```

or by

```
   1 D 0 90 180 270
```

Complex Algorithms

Many utility functions in *COMPLEX* are analogs of primitive APL computational and structural functions so it is often straightforward to convert a real number algorithm to one for complex numbers. Use functions prefixed with 'Z' to operate on complex singletons, and functions prefixed with 'CX' when dealing with arbitrary complex arrays.

For example, the following function finds all the proper factors of a real positive integer:

```
     ∇R←FACTORS N;C
[1]    ⍝ Find all proper factors of
[2]    ⍝   positive integer N > 2.
[3]    ⍝ Get candidates not 1 or N:
[4]    C←(2-⎕IO)+⍳N-2
[5]    ⍝ See which divide evenly:
[6]    R←(0=C|N)/C
       ∇
       FACTORS 20002
 2 73 137 146 274 10001
```

FACTORS can be modified for the complex case:

```
     ∇R←CXFACTORS N;A;C;I;K;M
[1]    ⍝ Find all proper complex factors of a
[2]    ⍝   complex integer N with magnitide > 1.
[3]    ⍝ Develop set of candidates:
[4]    A←ZMAG N ⍝ Magnitude limit
[5]    ⍝ Limit on each integer part:
[6]    K←⌊A÷2*0.5
[7]    ⍝ Get integers from -K to K:
[8]    R←⍴I←(-K+⎕IO)+⍳1+2×K
[9]    ⍝ Initial candidates are all complex
'10]   ⍝   integers in the square from
[11]   ⍝   (-K, -K) to (K,K):
[12]   ⍝ Construct all combinations:
[13]   C←(R/I) J (R*2)⍴I
[14]   ⍝ Magnitude test for "proper" ones:
[15]   M←CXMAG C ◇ C←((1<M)∧M<A)/C
[16]   ⍝ Finally see which divide evenly:
[17]   R←(∧/0=C CXRESIDUE N)/C
       ∇
       CXFACTORS 2 23
 ¯5 ¯4 ¯3 ¯2  2  3  4  5
  4  5 ¯2  3 ¯3  2  5 ¯4
```

Check that 4 5 is a factor of 2 23:

```
2 23 ZDIV 4 5
3 2
```

Complex Graphs

Not only will plotting the results of complex functions help you understand the properties of complex numbers, but it will also provide you with a large portfolio of fancy, easily generated graphs. First create this function to build a circle of points in the complex plane:

```
     ∇R←CXCIRCLE A;⎕IO
[1]    ⍝ Build vector of complex numbers
[2]    ⍝   representing circle in complex plane.
[3]    ⍝ A=radius, center (x,y), no. of points
[4]    ⍝   Default argument is 1, 0 0, 120
[5]    ⍝ The first point is repeated at the end
[6]    ⍝   for plotting purposes
[7]    ⎕IO←1 ◇ A←4↑,A
[8]    A←A+ 1 0 0 120 ×A=0
[9]    R←A[1] D (360÷A[4])×0,⍳A[4]
[10]   R←A[2 3] CXPLUS R
       ∇
```

Next, we need a function that maps a region in the complex plane to the graphics easel (with origin at upper left corner) used by the APL★PLUS System graphics primitives:

```
     ∇ C CXPLOT X;N;Y
[1]    ⍝ Plot complex numbers as polygonal line
[2]    ⍝   X=vector of complex numbers
[3]    ⍝   C=optional color(s)
[4]    ⍝ Maps complex rectangle from ¯10 ¯7.5
[5]    ⍝   to 10 7.5 onto 1024-by-1024 graphics
[6]    ⍝   easel with aspect ratio of 1.333
[7]    ⍝ Assumes graphics hardware already
[8]    ⍝   initialized, and return to text
[9]    ⍝   mode handled separately later
[10]   ⍝ Transpose data to 2 columns:
[11]   N←⍴Y←⍉CX X
[12]   ⍝ Transform complex region to easel:
[13]   Y←512+Y×N⍴ 51 ¯68
[14]   ⍝ Make default color 1:
[15]   ⍞(0=⎕NC 'C')/'C←1'
[16]   ⍝ Omit next line just to plot dots:
[17]   Y←(1,N)⍴Y
[18]   ⍝ Plot connecting lines:
[19]   C ⎕GLINE Y
       ∇
```

To produce a graph, let's create a circle with radius 2 and center at 0.5 0:

```
Z←CXCIRCLE 2 .5 0
```

and then plot it with the statement:

```
GINIT ◇ CXPLOT Z ◇ GEND
```

where *GINIT* and *GEND* are expressions or functions to initialize and exit graphics mode, respectively. For some hardware configurations, *GINIT* might contain

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```
0 ⎕GINIT 'IBMCOLOR'
```

and GEND might contain

```
0⍴⎕INKEY ◇0 0⍴3 ⎕INT 16
```

Just plotting Z produces the expectedly tame circle, but your graphics are soon enlivened when you plot various complex functions of Z. Try

```
GINIT ◇ CXPLOT Z ◇ 2 CXPLOT .1×Z CXSTAR 5 ◇ GEND
```

to get an intricate plot that illustrates how raising to the fifth power transforms a circle. Additional examples that produce some interesting graphs are:

```
CXSIN Z                    CXTAN Z
CXSINH Z                   CXTANH Z
CXACOSH Z                  CXATANH Z
2 × CXLOG Z                .5×CXEXP Z
CXSIN Z CXPOWER 1.75       3×CXTANH CXTAN Z
Z CXTIMES CXSIN Z          (CXEXP Z) CXDIV Z
CXFLOOR Z                  4 × 2 1 CXRESIDUE Z
```

To produce a scatterplot instead of a line plot, you can comment out line 17 of *CXPLOT*. Try that, and then plot

```
CXFACTORS 15
```

to see the geometric symmetries among all the complex factors of 15.

APL★PLUS FOR THE IBM RT PC SCHEDULED FOR MARCH RELEASE

STSC is putting the final touches on its APL★PLUS UNX System for the IBM RT PC under AIX.

The IBM RT PC is IBM's first venture into RISC (Reduced Instruction Set Computing) and also its first major entry into the world of UNIX computing. This computer has received very favorable reviews in the press and seems certain to play a large role in the UNIX community.

The APL★PLUS System for UNIX environments is a state-of-the-art implementation of APL running on supermicros and minicomputers. It contains all the standard APL★PLUS enhancements to APL (component file system, report formatter, and other system functions), as well as some powerful new features:

- a ring editor that allows editing of multiple objects
- access to non-APL programs and facilities, including forked processes (multi-tasking)
- virtually unlimited workspace size and object size
- nested arrays
- true shared file system
- precompilation of user-defined APL functions.

The APL★PLUS System is currently available for the following machines running UNIX:

AT&T 3B series, DEC VAX (Ultrix), HP 9000/300 and 9000/500, IBM PC AT (Xenix), NCR Tower, Sequent Balance, and Sun Workstation.

To find out more about the APL★PLUS UNX System for the IBM RT PC, contact STSC at 2115 East Jefferson Street, Rockville, MD 20852, or call (800) 592-0050 in the U.S. In Maryland, Canada, or outside the U.S. call (301) 984-5112.

STATGRAPHICS TRAINING

Statistical Graphics Corporation, the developers of STATGRAPHICS, will continue offering a series of 2½ day seminars in 1987 in Princeton, New Jersey. The course has been divided into two separate classes as listed below.

STATGRAPHICS for Quality Control Professionals	**Interactive Data Analysis Using STATGRAPHICS**
April 6-8	April 8-10
July 20-22	July 22-24
October 19-21	October 21-23
Monday & Tuesday	**Wednesday**
8:45 A.M. - 5:00 P.M.	1:00 P.M. - 5:00 P.M.
Wednesday	**Thursday & Friday**
8:45 A.M. - 12:30 P.M.	8:45 A.M. - 5:00 P.M.

Fee for each seminar is $650, with discounts offered for companies sending more than one person. Other seminars are available to be held at your company's site. To register or for more information, call Statistical Graphics Corporation at (609) 924-9374.

FREE UPGRADE RELEASED FOR STATGRAPHICS SYSTEM

Registered users of STATGRAPHICS Version 2.0 will be receiving the latest upgrade, Version 2.1, free of charge.

The Version 2.1 upgrade includes improved behavior in several of the procedures and corrections to edge conditions reported by Version 2.0 users. Improvements in Version 2.1 include:

- Improved keyboard support.
- Support for Version 6.1 of STSC's APL★PLUS System for the PC. The APL★PLUS System may be used with STATGRAPHICS by analysts who wish to write customized procedures.
- The addition of a mathematical operator for calculating derivatives.
- Improvements to the plotter and printer interfaces, including the ability to interrupt output before completion.
- Improvements in the chi-square test provided with the distribution fitting procedures, the canonical correlations procedure, the Tukey post hoc test provided with the analysis of variance procedures, and the import data files procedures.
- General improvements in error handling throughout the system.
- More efficient handling of integer data in multiple regression, analysis of variance, and other procedures.

Users who have not registered their Version 2.0 STATGRAPHICS System can receive the Version 2.1 upgrade by completing and mailing the registration card that came with their Version 2.0 system along with the label from their Start-Up Disk. If you prefer to send in your original Start-Up Disk, be sure to make a back up copy so that you can continue to use STATGRAPHICS. Users of earlier versions of STATGRAPHICS can also receive the newest release by paying an upgrade charge.

For more information, contact STSC, 2115 East Jefferson Street, Rockville, MD 20852, ATTN: STATGRAPHICS Upgrade, or call (301) 984-5123. International customers should contact the dealer they purchased Version 2.0 from for their free update.